Spin polarised nuclear matter and its application to neutron stars

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Abstract

An equation of state(EOS) of nuclear matter with explicit inclusion of a spin-isospin dependent force is constructed from a finite range, momentum and density dependent effective interaction. This EOS is found to be in good agreement with those obtained from more sophisticated models for unpolarised nuclear matter. Introducing spin degrees of freedom, it is found that at density about 2.5 times the density of normal nuclear matter the neutron matter undergoes a ferromagnetic transition. The maximum mass and the radius of the neutron star agree favourably with the observations. Since finding quark matter rather than spin polarised nuclear matter at the core of neutron stars is more probable, the proposed EOS is also applied to the study of hybrid stars. It is found using the bag model picture that one can in principle describe both the mass and size as well as the surface magnetic field of hybrid stars satisfactorily.

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1. Introduction

Since the pioneering works of Baade and Zwicky[1] and Oppenheimer and Volkoff[2] about half a century ago, and the identification of pulsars as rotating neutron stars around the year 1968[3], several studies have been made to unravel the mystery of the structure of the neutron stars. It is a well accepted fact that the density inside a neutron star varies from the surface to the core by about 15 orders of magnitude. Understanding the structure of such complex objects requires an accurate knowledge of the equation of state(EOS) of neutron star matter in the different density regions. The extremely low density domain and the subnuclear region can be well described by the EOS given by Feynmann-Metropolis-Teller (FMT)[4] and Baym-Pethick-Sutherland (BPS)[5] respectively. The EOS of the dense nuclear matter is still riddled with some uncertainties. A consistent EOS should describe the compressional properties[6] of nuclei near the ground state and also of the hot and dense nuclear material that is created in energetic nuclear collisions[7]. It should also answer such important questions as whether a star at the later stage of its life explodes or not[8] and how the neutron star is born. Renewed interest in the rapid cooling of neutron stars[9, 10] by the direct URCA process has also an immediate bearing on the nuclear EOS.

In the non-relativistic framework, EOS of nuclear matter have been constructed with phenomenological effective interactions or from realistic interactions with different degrees of sophistication[11-16]. Most of these EOS can explain the properties of neutron stars like their mass, size, moment of inertia etc. well within the observational limits. However, in regard to the magnetic properties of neutron stars, till now no acceptable explanation exists for the origin of the rather large magnetic field ($\sim 10^{12}G$) at the surface. A recent analysis[17] of binary millisecond pulsars suggests that a permanent component of this magnetic field could exist, sustained by a spontaneous magnetised phase inside the neutron star. Attempts have been made to explain the

presence of the magnetic field by means of a ferromagnetic transition.

In the framework of Hartree-Fock theory, employing hard and also soft core potentials, Pfarr[18] does not get such a ferromagnetic transition. A similar conclusion is reached by Forseth and Ostgaard[19] who made the calculations in the lowest order constrained variational method of Pandhari- pande using soft core potentials; with a hard core potential a transition to the ferromagnetic state was seen to occur though at $\sim 30\ \rho_o$, where ρ_o is the normal nuclear matter density. In a relativistic $\sigma + \omega$ Hartree-Fock approach, a ferromagnetic transition is also predicted by Niembro et al[20] at too high a density. However, in an improved model[21] with inclusion of π and ρ mesons in addition to $\sigma + \omega$, a ferromagnetic transition is seen to occur at a compar- atively much lower density, $\rho \sim 3.5\rho_o$, but the incompressibility of normal nuclear matter is found to be too high (~ 450 MeV). It would therefore be interesting to know whether a ferromagnetic phase transition is possible at a density realisable in neutron star matter with an EOS with firmer grounds in the experimental realities of normal nuclear matter and finite nuclei. This is the primary motivation of this work.

There is a strong possibility that at the core densities of neutron stars, there is a phase transition from nuclear matter to quark matter[22-25]. About a decade ago, Witten conjectured[26] that the strange quark matter(SQM) might be the absolute ground state of hadronic matter i.e., the mass energy per baryon may be less than 930 MeV. If this is true, then the possibility that the pulsars are rotating strange quark stars may not be ruled out. Even if SQM is not the absolute ground state (i.e. at densities less than the hadron- quark transition density the nuclear matter is energetically favoured than SQM), one may still find hybrid stars having quark cores with nucleon en- velops. Since our understanding of the confinement/deconfinement of quarks is far from complete, all the aforesaid possibilities are only speculative. One should also keep in mind that the theoretical framework used in general to study the phase transition is phenomenological and simplistic in nature.

In view of all the above, we would like to investigate in this paper the

following. Firstly, whether there exists an EOS which consistently describes the nuclear matter and finite nuclear properties, as well as predict a ferromagnetic transition at a density realisable in the interior of neutron stars. Secondly, whether the same EOS which predicts a ferromagnetic transition permits a hadron-quark phase transition and thereby the formation of a hybrid star. And finally using such an EOS, whether we can consistently explain the structural properties such as mass, radius and moment of inertia as well as the presence of the magnetic field at the surface within the same model.

2. Theoretical framework

In the following, we briefly outline the procedure to obtain the nuclear equation of state in a non-relativistic framework and discuss its merits and limitations.

2.1. Equation of state

The phenomenological momentum and density dependent finite range interaction employed here to obtain the equation of state is a modified version of Seyler-Blanchard interaction[27]. To treat spin-polarised isospin asymmetric nuclear matter, the interaction has been generalised to include explicitly the spin-isospin dependent channel. The interaction between two nucleons with separation r and relative momentum p is given by,

$$v_{eff}(r,p,\rho) = -C_{\tau s}\left[1 - \frac{p^2}{b^2} - d^2(\rho_1(r_1) + \rho_2(r_2))^n\right]\frac{e^{-r/a}}{r/a} \ , \qquad (1)$$

where a is the range and b defines the strength of the repulsion in the momentum dependence of the interaction. The parameters d and n are measures of the strength of the density dependence, and ρ_1 and ρ_2 are the densities at the sites of the two interacting nucleons. The subscripts τ and s in the strength parameter $C_{\tau s}$ refer to the likeness l and the unlikeness u in the isotopic spin and spin of the two nucleons respectively; for example, C_{ll} refers to interactions between two neutrons or protons with parallel spins, C_{lu} refers to that

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between neutrons or protons with opposite spin *etc.* The energy per nucleon E and the pressure P in the mean-field approximation can then be worked out[27] as

$$E = \frac{1}{\rho} \sum_{\tau s} \rho_{\tau s} \left[T \frac{J_{3/2}(\eta_{\tau s})}{J_{1/2}(\eta_{\tau s})} (1 - m^*_{\tau s} V^1_{\tau s}) + \frac{1}{2} V^0_{\tau s} \right] , \tag{2}$$

$$P = \sum_{\tau s} \rho_{\tau s} \left[\frac{2}{3} T \frac{J_{3/2}(\eta_{\tau s})}{J_{1/2}(\eta_{\tau s})} + V^0_{\tau s} + \frac{1}{2} b^2 \left(1 - d^2 (2\rho)^n \right) V^1_{\tau s} + V^2_{\tau s} \right] . \tag{3}$$

Here, $J_k(\eta)$ are the Fermi integrals, $V^0_{\tau s}$ and $V^2_{\tau s}$ are the single-particle and the rearrangement potentials, $V^1_{\tau s}$ is the coefficient of the quadratic momentum dependent term in the potential and defines the effective mass $m^*_{\tau s}$, T the temperature, and η is the fugacity given by $\eta_{\tau s} = (\mu_{\tau s} - V^0_{\tau s} - V^2_{\tau s})/T$. For the unpolarised nuclear matter(NM), the expressions for $V^0_{\tau s}$ *etc* are given in ref.[27]. It is straightforward to extend these to the case of polarised nuclear matter and are as given below:

$$
\begin{aligned}
V^0_{\tau s} &= -4\pi a^3 \left(1 - d^2 (2\rho_o)^n \right) \left[C_{ll} \rho_{\tau,s} + C_{lu} \rho_{\tau,-s} + C_{ul} \rho_{-\tau,s} + C_{uu} \rho_{-\tau,-s} \right] \\
&\quad + \frac{8\pi^2 a^3}{b^2 h^3} \{ C_{ll} (2m^*_{\tau,s} T)^{5/2} J_{3/2}(\eta_{\tau,s}) + C_{lu} (2m^*_{\tau,-s} T)^{5/2} J_{3/2}(\eta_{\tau,-s}) \\
&\quad + C_{ul} (2m^*_{-\tau,s} T)^{5/2} J_{3/2}(\eta_{-\tau,s}) + C_{uu} (2m^*_{-\tau,-s} T)^{5/2} J_{3/2}(\eta_{-\tau,-s}) \}, \\
V^1_{\tau s} &= \frac{4\pi a^3}{b^2} \left[C_{ll} \rho_{\tau,s} + C_{lu} \rho_{\tau,-s} + C_{ul} \rho_{-\tau,s} + C_{uu} \rho_{-\tau,-s} \right], \\
V^2_{\tau s} &= 4\pi a^3 d^2 n (2\rho_o)^{n-1} \sum_{\tau',s'} \left[C_{ll} \rho_{\tau',s'} + C_{lu} \rho_{\tau',-s'} + C_{ul} \rho_{-\tau',s'} + C_{uu} \rho_{-\tau',-s'} \right] \rho_{\tau',s'}, \\
m^*_{\tau s} &= \left[\frac{1}{m_\tau} + 2 V^1_{\tau s} \right]^{-1}. \tag{4}
\end{aligned}
$$

One usually defines the neutron and proton spin excess parameters (spin asymmetry) as

$$
\begin{aligned}
\alpha_n &= (\rho_{n\uparrow} - \rho_{n\downarrow})/\rho , \\
\alpha_p &= (\rho_{p\uparrow} - \rho_{p\downarrow})/\rho , \tag{5}
\end{aligned}
$$

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where

$$\rho = \rho_n + \rho_p = (\rho_{n\uparrow} + \rho_{n\downarrow}) + (\rho_{p\uparrow} + \rho_{p\downarrow}) , \qquad (6)$$

is the number density. We then define the proton fraction as $x = \rho_p/\rho$. It is related to the isospin asymmetry parameter X as,

$$X = (1 - 2x) = (\rho_n - \rho_p)/\rho . \qquad (7)$$

We also define the spin excess parameter as $Y = \alpha_n + \alpha_p$ and the spin-isospin excess parameter as $Z = \alpha_n - \alpha_p$. One can then express the energy per nucleon E/A of the NM at zero temperature as

$$E/A = E_V + E_X X^2 + E_Y Y^2 + E_Z Z^2 , \qquad (8)$$

where terms higher than those quadratic in X, Y and Z are neglected. Here E_V is the volume energy of the symmetric nuclear matter, taken as -16.1 MeV and E_X is the usual symmetry (isospin) energy, taken to be 34.0 MeV. The quantities E_Y and E_Z are the spin and the spin-isospin symmetry energies of the NM respectively. Their values are uncertain to some extent. We take[28, 29] $E_Y = 31.5$ MeV and $E_Z = 35.0$ MeV in conformity with the generalised hydrodynamical model of Uberall[30], where $(E_Z/E_X)^{1/2} \simeq 1.1$. In terms of the strength parameters $C_{\tau s}$, the volume and the symmetry energies are written in the form,

$$\begin{pmatrix} -A & -A & -A & -A \\ -B & -B & C & C \\ -B & C & -B & C \\ -B & C & C & -B \end{pmatrix} \begin{pmatrix} C_{ll} \\ C_{lu} \\ C_{ul} \\ C_{uu} \end{pmatrix} = \begin{pmatrix} E_V - 3p_F^2/(10m) \\ E_X - p_F^2/(6m) \\ E_Y - p_F^2/(6m) \\ E_Z - p_F^2/(6m) \end{pmatrix} . \qquad (9)$$

Here p_F is the Fermi momentum of the one-component nuclear matter corresponding to the density ρ of the polarised NM, $A = \alpha(\beta - \delta)$, $B = \alpha(\beta - 20\delta/9)$, $C = \alpha(\beta - 10\delta/9)$, $\alpha = 8\pi^2 a^3 p_F^3/(3h^3)$, $\beta = 1 - d^2(2\rho)^n$ and $\delta = 6p_F^2/(5b^2)$. For a fixed value of n, the parameters $C_{\tau s}$, a, b and d are then determined by reproducing E_V, E_X, E_Y, E_Z, the saturation density

of normal nuclear matter($\rho_0 = 0.1533 \; fm^{-3}$), the surface energy coefficient ($a_S = 18.0$ MeV), and the energy dependence of the real part of the nucleon-nucleus optical potential. The parameter n is determined by reproducing the breathing-mode energies[31].

The parameters of the interaction are listed below:

$$
\begin{aligned}
C_{ll} &= -305.2 \text{ MeV} & a &= 0.625 \text{ fm} \\
C_{lu} &= 902.2 \text{ MeV} & b &= 927.5 \text{ MeV/c} \\
C_{ul} &= 979.4 \text{ MeV} & d &= 0.879 \text{ fm}^{3n/2} \\
C_{uu} &= 776.2 \text{ MeV} & n &= 1/6.
\end{aligned}
$$

With the above value of the parameter n, the incompressibility of symmetric nuclear matter is $K = 240$ MeV.

2.2. *Merits and limitations*

It has been tested that the above interaction reproduces quite well the ground state binding energies, root mean square charge radii, charge distributions and giant monopole resonance energies for a host of even-even nuclei ranging from ^{16}O to very heavy systems. Interactions of this type have been used before with great success by Myers and Swiatecki[32] in the context of nuclear mass formula. We have also seen that for symmetric nuclear matter, our results agree extremely well with those calculated in a variational approach by Friedmann and Pandharipande(FP)[12] with $v_{14} + TNI$ interaction in the density range $\frac{1}{2}\rho_0 \leq \rho \leq 2\rho_0$. However, for unpolarised pure neutron matter, the energies calculated with our interaction are somewhat higher compared to the FP energies, particularly at higher densities. The entropy per particle for neutron matter calculated with our interaction at different temperatures agrees extremely well with the corresponding FP results. In Fig.1, the energy per particle for neutron matter is displayed as a function of density at zero temperature. For comparision, the FP energies[12]

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and those obtained with the Bethe-Johnson(BJ) potential[33] in a sophisticated correlated basis function approach are also displayed. The BJ curve is very close to ours for neutron matter. This good agreement between our calculations and those reported in refs.[12] and [33] suggests that the present interaction can be extrapolated with some confidence to neutron matter or to nuclear matter with large isospin asymmetry at high densities. It can also be mentioned that such an interaction satisfies the Landau-Migdal stability criteria[34].

All the well-known non-relativistic nuclear equations of state suffer from lack of causality at high densities. The velocity of sound in nuclear matter then becomes superluminal. The effective interaction used by us is no exception. In Fig.2, we have plotted the velocity of sound in units of c as a function of the ratio ρ/ρ_o in the case of neutron matter taking $\alpha_n = 0$, 0.3, and 0.5. It can be seen that as spin-polarisation increases, the EOS becomes softer and the velocity of sound v_s becomes acausal only at increasingly higher densities. This superluminous behaviour of v_s, particularly for the unpolarised neutron matter, suggests that the extrapolation of such an EOS to very high nuclear densities may not be advisable.

3. Ferromagnetic phase transition

It would be interesting to investigate whether the nuclear EOS discussed in the previous section, in addition to the consistent description of the nuclear matter and finite nuclear properties predicts a ferromagnetic transition at densities meaningful in the context of neutron stars.

It has been conjectured[35, 36] that in the neutron star matter, in contrast to nuclei where the neutrons generally pair up to spin $J = 0$ in their ground states, the neutrons may pair up to spin $J = 1$ at higher nuclear densities, thus leading to a ferromagnetic transition. To investigate this aspect, we calculate the energy per particle for the spin polarised neutron matter with a representative value of $\alpha_n = 0.5$ and compared with that calculated for

unpolarised neutron matter in Fig.1. We find that above $\rho \sim 2.5\rho_o$, the energy of polarised matter is lower compared to that for unpolarised neutron matter. This reflects that the neutrons that pair up to $J = 0$ at lower densities undergo a transition to a spin polarised configuration as density builds up. In otherwords, the system prefers a ferromagnetic state for $\rho > 2.5\rho_o$.

The behaviour of the magnetic susceptibility χ of neutron matter as a function of density also portrays the occurence of ferromagnetic phase transition. In general, the magnetic susceptibility is defined[37] as $\chi = \frac{\partial M}{\partial H}$, where H is the magnetic field and $M = \mu_n(N_\uparrow - N_\downarrow)/V$ is the total magnetisation per unit volume with μ_n being the magnetic moment of a neutron. Using the definition of α_n[Eq.(5)], we rewrite M as $M = \mu_n\alpha_n\rho_n$.

We need to determine the optimum value of α_n using the energy minimisation criteria,

$$\frac{\partial(E_H(\rho, \alpha_n)/N)}{\partial\alpha_n} \Big|_{\alpha_n=\alpha_n^0} = 0. \tag{10}$$

The total energy E_H/N per particle of a system of N number of neutrons in the presence of an external weak magnetic field H is,

$$E_H(\rho, \alpha_n)/N = E(\rho, \alpha_n)/N - (\mu_n H \alpha_n). \tag{11}$$

Expanding the energy $E(\rho, \alpha_n)$ in powers of α_n upto $O(\alpha_n^2)$, we get

$$
\begin{aligned}
E(\rho, \alpha_n)/N &= E(\rho, \alpha_n = 0)/N + \frac{1}{2}\alpha_n^2\frac{\partial^2(E(\rho, \alpha_n)/N)}{\partial\alpha_n^2} \Big|_{\alpha_n=0}, \\
&\equiv e_0 + \frac{1}{2}\alpha_n^2 e_2.
\end{aligned}
\tag{12}
$$

Because the energy $E(\rho, \alpha_n)$ is symmetric in α_n, all the odd derivatives in the expansion of $E(\rho, \alpha_n)$ vanish. (It may be said here that only in the calculation of χ, we have expanded $E(\rho, \alpha_n)$ in powers of α_n, otherwise we have calculated it numerically.) Then, the optimum value α_n^0 is determined by minimising the energy[Eq.(11)] as $\alpha_n^0 = \mu_n H/e_2$. Now, we can determine

χ and it is given as,

$$\chi = \frac{\partial}{\partial H}\left(\mu_n \alpha_n^0 \rho_n\right) = \frac{\mu_n^2 \rho_n}{e_2}. \tag{13}$$

Using the effective interaction given in Eq.(1), we get

$$
\begin{aligned}
e_2 &= \frac{\partial^2 (E(\rho,\alpha_n)/N)}{\partial \alpha_n^2}\Big|_{\alpha_n=0}, \\
&= -2\pi a^3 \rho_n \left[A_1 (C_{ll} - C_{lu}) - \frac{20\ 2^{2/3}}{9}\ A_2 (2C_{ll} - C_{lu}) \right] + \frac{2^{2/3} p_F^2}{3m}, \tag{14}
\end{aligned}
$$

where $A_1 = 1 - d^2(2\rho_n)^n$ and $A_2 = \rho_n p_F^3/b^3$. In the limit of no interaction, e_2 is simply given by the kinetic term alone, i.e. $e_2^{free} = 2^{2/3} p_F^2/(3m)$. It is then straightforward to calculate the ratio χ_{free}/χ, where χ_{free} is the magnetic susceptibility of the non-interacting neutron gas. The onset of a ferromagnetic transition is depicted by the vanishing of the ratio χ_{free}/χ. Further, the effect of the nuclear matter incompressibility K on the ferromagnetic transition density is studied and the results are shown in Fig.3. As the value of K is increased from 240 MeV to 304 MeV (by increasing the density exponent n of the effective interaction given by eq.(1) from 1/6 to 2/3), the density at which the transition takes place decreases from $\sim 2.4\rho_o$ to $\sim 2.3\rho_o$. It is thus seen that the effective interaction given in Eq.(1) predicts a ferromagnetic phase transition at a density $\rho \sim 2.4\rho_o$.

In previous calculations in the non-relativistic formalism using realistic interactions, one usually does not find such a ferromagnetic transition [18, 19] or if there is such a possibility, it occurs at densities[19] not realisable in the context of neutron stars. In a relativistic framework, one finds that the simple $\sigma + \omega$ model[20] does not suggest any such transition. However, with an improved model[21] including other mesons like ρ and π in addition to $\sigma + \omega$, one finds a transition at about $\rho \sim 3.5\rho_o$. But, the incompressibility of the normal nuclear matter obtained in this model is too high ($\sim 450\ MeV$). In contrast, our nuclear interaction that predicts a ferromagnetic transition at a similar density yields a value of incompressibility that is close to the one

well-accepted[6].

4. Structural properties of neutron stars

In this section, we explore the various static properties of neutron stars such as proton fraction, mass, size and moment of inertia using the proposed equation of state. We further study the influence of spin polarisation on these observables.

4.1. *Beta-equilibrium proton fraction*

In recent years, attention has been drawn to the direct URCA process in neutron stars which may be the primary mechanism for its rapid cooling. This can, however, occur only when the beta-equilibrium proton fraction x in the star is ≥ 0.11, where only electrons are considered, and ≥ 0.148, if both electrons and muons are considered. It would be interesting to know whether spin polarisation favours or disfavours direct URCA process. In our study, the lepton energy per particle $E_L(\rho, x)$ is given by the relativistic, ideal Fermi-gas expression[38]; in addition to e^-, μ^- are also considered as and when they are energetically favoured. At beta-equilibrium, one has $\frac{\partial}{\partial x}(E(\rho, x) + E_L(x)) = 0$, where $E(\rho, x)$ is the baryonic energy per particle including the rest masses. In Fig.4, the beta-equilibrium proton fraction thus obtained in the neutron star matter is displayed as a function of the baryon density invoking the condition of charge neutrality. The upper panel corresponds to unpolarised matter, and the lower panel displays that for spin polarised matter with $\alpha_n = 0.3$ and $\alpha_n = 0.5$. From the upper panel, it can be seen that for $\alpha_n = 0$, x shows a peaked structure against density. When only e^- are considered, x increases as density increases, reaches a maximum value of about 0.085 at $\rho \simeq 3\rho_o$, and then decreases to very low values at higher densities. With the inclusion of μ^-, the structure of the curve remains almost the same; however, it lies higher than that of the former case, at all densities.

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The peak value is then about 0.11. The lower panel of Fig.4 displays proton fractions for the spin polarised neutron star matter with $\alpha_n = 0.3$ and $\alpha_n = 0.5$. Here both muons and electrons are taken into consideration for calculating proton fractions at beta-equilibrium. With increasing spin polarisation, the proton fraction becomes smaller at any density. It may be noted that the present EOS in use does not favour direct URCA process, since the proton fraction is always below the critical value. Introduction of spin polarisation disfavours direct URCA process even more. It may be remarked that various calculations give different conclusions[10] regarding the direct URCA process. It may also be mentioned that inclusion of exotic processes like pion condensation or kaon condensation in dense neutron star matter may enhance the proton fraction thereby favouring direct URCA process[39], but occurence of such exotic phenomena is still very much unsettled[9].

4.2. *Mass and size of neutron stars*

We now determine the structure of neutron stars using a composite EOS i.e. FMT, BPS, Baym-Bethe-Pethick(BBP)[11] and the present interaction with progressively increasing densities. Then, the total mass and the size of the neutron star can be obtained by solving the general relativistic Tolman-Oppenheimer-Volkoff(TOV) equation,

$$\frac{dP(r)}{dr} = -\frac{G}{c^4} \frac{[\epsilon(r) + P(r)]\,[m(r)c^2 + 4\pi r^3 P(r)]}{r^2 \left[1 - \frac{2Gm(r)}{rc^2}\right]}, \tag{15}$$

where,

$$m(r)c^2 = \int_0^r \epsilon(r')d^3r'. \tag{16}$$

The quantities $\epsilon(r)$ and $P(r)$ are the energy density and pressure at a radial distance r from the centre, and are given by the equation of state. The mass of the star contained within a distance r is given by $m(r)$. The size of the star is determined by the boundary condition $P(R) = 0$ and the total mass M of the star integrated upto the surface R is given by $M = m(R)$. The single

integration constant needed to solve the TOV equation is P_c, the pressure at the center of the star calculated at a given central density ρ_c.

The mass functions of the star thus obtained as a function of its central density are shown in Fig.5 for four different values of spin polarisation ($\alpha_n = 0.0,\ 0.3,\ 0.4,\ 0.5$). The radii, central densities and surface redshifts corresponding to the maximum mass M_{max} configuration are tabulated in Table 1 for three values of α_n. The surface redshift z_s is defined[11] as

$$z_s = \left[1 - \frac{2GM}{Rc^2}\right]^{-1/2} - 1. \tag{17}$$

Values of $M_{max}/M_\odot$ are also given in the same table. It can be seen that the maximum mass and the corresponding radius and surface redshift z_s decrease with increasing polarisation. On the other hand, the central density pertaining to M_{max} configuration increases as α_n increases. This is because of the fact that the spin polarised neutron star matter is more compressible than the unpolarised one. The measured mass[40] of $4U0900-40$, $(1.85\pm0.3)M_\odot$ possibly provides the lower limit of the maximum mass of the neutron star. Our calculations with the values of α_n taken are well within this limit. The maximum mass of neutron star obtained from calculations with $\alpha_n = 0.6$ and above are found to be below the present observational limits and hence we have restricted our calculations upto $\alpha_n = 0.5$. It may be mentioned that the unpolarised neutron matter is close to being superluminal at the central density corresponding to the maximum mass. For polarised neutron star matter though the central density increases significantly, the sound velocity is always luminal because of the softness of the polarised matter towards compression.

We have also studied the sensitivity of the mass distribution $m(r)$ to α_n, where $m(r)$ denotes the total mass contained within a given radial distance r from the centre. Fig.6 displays $m(r)/M_{max}$ as a function of the density $\rho(r)$ at that point r, where M_{max} corresponds to the maximum mass configuration at a given α_n. The zero of the abscissa refers to the surface of the neutron

star; the points where the mass functions $m(r)$ meet the abscissa refer to the centre of the star. It is found that nearly 90% of the mass of the neutron star lies at densities higher than $3\rho_o$.

4.3. *Moment of inertia of neutron stars*

The moment of inertia of neutron stars is calculated by assuming the star to be rotating slowly with an uniform angular velocity Ω[41]. The angular velocity $\bar{\omega}(r)$ of a point in the star measured with respect to the angular velocity of the local inertial frame is determined by the equation,

$$\frac{1}{r^4}\frac{d}{dr}\left[r^4 j \frac{d\bar{\omega}}{dr}\right] + \frac{4}{r}\frac{dj}{dr}\bar{\omega} = 0, \qquad (18)$$

where,

$$j = e^{-\phi(r)}\left(1 - \frac{2Gm(r)}{rc^2}\right)^{1/2}. \qquad (19)$$

The function $\phi(r)$ is constrained by the condition,

$$e^{\phi(r)}\mu(r) = \text{constant} = \mu(R)\sqrt{1 - \frac{2GM}{Rc^2}}, \qquad (20)$$

where the chemical potential $\mu(r)$ is defined as,

$$\mu(r) = \frac{\epsilon(r) + P(r)}{\rho(r)}. \qquad (21)$$

Using these relations, Eq.(18) can be solved subject to the boundary conditions that $\bar{\omega}(r)$ is regular as $r \longrightarrow 0$, and $\bar{\omega}(r) \longrightarrow \Omega$ as $r \longrightarrow \infty$. Then moment of inertia of the star can be calculated using the definition $I = J/\Omega$, where the total angular momentum J is given as

$$J = \frac{c^2}{6G} R^4 \left(\frac{d\bar{\omega}}{dr}\right)|_{r=R} . \qquad (22)$$

Values of I thus obtained for three values of α_n are plotted as a function of the central density in Fig.7. It can be seen that the maximum value of I

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greatly depends on α_n. As α_n is increased from 0.0 to 0.5, I_{max} has decreased by about 50%.

We have thus calculated the important structural properties of neutron stars and studied their dependence on the spin polarisation factor. It is found that upto a value $\alpha_n = 0.5$, the properties obtained here are well within the observational limits. Motivated by this success in explaining the structural properties of neutron stars, we are curious to see how far we can account for the surface magnetic field. Our present study suggests that the neutron star matter at densities $\rho > 2.5\rho_o$ is spin polarised. Taking $\alpha_n = 0.5$, one immediately realises that the surface magnetic field is largely overestimated($\sim 10^{16}G$). As noted earlier, it is probable that that at core densities one finds quark matter rather than spin polarised nuclear matter. Therefore, we in the following section explore this plausibility and its implications upon both the structural and magnetic properties of stars.

5. Hybrid stars

Here, we construct the $\beta-$ equilibrated, electrically neutral quark matter equation of state and employ it to understand the structural properties of the hybrid stars,i.e. quark cores with nucleon envelopes.

The equation of state of a three flavour quark matter consisting u, d and s quarks is obtained here using the phenomenological MIT bag model[42]. The total kinetic energy density of a system of non-interacting, relativistic quarks of flavour τ and mass m_τ is given as,

$$\epsilon_\tau = \frac{3}{8\pi^2} \frac{(m_\tau c^2)^4}{(\hbar c)^3} \left[x_\tau \sqrt{1 + x_\tau^2} \left(1 + 2x_\tau^2\right) - ln(x_\tau + \sqrt{1 + x_\tau^2}) \right], \qquad (23)$$

where $x_\tau = p_F^\tau/(m_\tau c)$, p_F^τ being the Fermi momentum and is related to the quark number density ρ_τ of a given flavour as $p_F^\tau = \hbar(\pi^2\rho_\tau)^{1/3}$. The densities pertaining to the three flavours can be expressed in terms of the total quark

number density ρ_q and the asymmetry parameters δ_{ud} and δ_{us} as:

$$\begin{aligned}
\rho_u &= (\rho_q/3)\left[1 - \delta_{ud} - \delta_{us}\right], \\
\rho_d &= (\rho_q/3)\left[1 + 2\delta_{ud} - \delta_{us}\right], \\
\rho_s &= (\rho_q/3)\left[1 - \delta_{ud} + 2\delta_{us}\right],
\end{aligned} \tag{24}$$

where $\delta_{ud} = (\rho_d - \rho_u)/\rho_q$, $\delta_{us} = (\rho_s - \rho_u)/\rho_q$ and $\rho_q = \rho_u + \rho_d + \rho_s$. Similarly, the energy density ϵ_L pertaining to a system of relativistic non-interacting electron gas can be calculated[38].

We then determine the equilibrium composition of the quark matter subject to the $\beta-$ equilibrium condition,

$$\mu_d - \mu_u = \mu_e \quad \text{and} \quad \mu_d = \mu_s, \tag{25}$$

and the charge neutrality condition,

$$\rho_e = \frac{1}{3}\left(2\rho_u - \rho_d - \rho_s\right). \tag{26}$$

Using Eq.(24) one obtains, $\rho_e = -(\rho_q/3)\,(\delta_{ud} + \delta_{us})$. Similarly, we can express the chemical potentials μ_u, μ_d and μ_s in terms of the three quantities ρ_q, δ_{ud} and δ_{us} as follows:

$$\begin{aligned}
\mu_u &= \left(\frac{\partial \epsilon_q}{\partial \rho_u}\right)_{|\rho_d,\rho_s} = \frac{\partial \epsilon_q}{\partial \rho_q} - \frac{1 + \delta_{ud}}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{ud}} - \frac{1 + \delta_{us}}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{us}}, \\
\mu_d &= \left(\frac{\partial \epsilon_q}{\partial \rho_d}\right)_{|\rho_u,\rho_s} = \frac{\partial \epsilon_q}{\partial \rho_q} + \frac{1 - \delta_{ud}}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{ud}} - \frac{\delta_{us}}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{us}}, \\
\mu_s &= \left(\frac{\partial \epsilon_q}{\partial \rho_s}\right)_{|\rho_u,\rho_d} = \frac{\partial \epsilon_q}{\partial \rho_q} - \frac{\delta_{ud}}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{ud}} + \frac{1 - \delta_{us}}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{us}},
\end{aligned} \tag{27}$$

where $\epsilon_q = \sum_\tau \epsilon_\tau + B$ is the total quark energy density and B is the bag parameter. Using these expressions, the $\beta-$ equilibrium conditions can be rewritten as,

$$\begin{aligned}
\frac{\partial \epsilon_q}{\partial \delta_{ud}} - \frac{\partial \epsilon_q}{\partial \delta_{us}} &= 0, \\
\frac{2}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{ud}} + \frac{1}{\rho_q}\frac{\partial \epsilon_q}{\partial \delta_{us}} &= \mu_e,
\end{aligned} \tag{28}$$

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where $\mu_e = \sqrt{p_{F,e}^2 c^2 + m_e^2 c^4}$. Thus, for a given baryon density $\rho_b = \rho_q/3$, the three quantities ρ_e, δ_{ud} and δ_{us} are fixed by the Eqs.(26) and (28). Subsequently, the equation of state is completely described by the total energy density ϵ_{QM} and the pressure P_{QM} of the system calculated for a given ρ_b using the definitions,

$$\epsilon_{QM} = \sum_\tau \epsilon_\tau(\rho_q, \delta_{ud}, \delta_{us}) + \epsilon_L(\rho_q, \delta_{ud}, \delta_{us}) + B,$$

$$P_{QM} = \rho_q \frac{\partial \epsilon_{QM}}{\partial \rho_q} - \epsilon_{QM}. \tag{29}$$

It may be mentioned that in our present study we have omitted the lowest order quark-quark interaction terms as it can be effectively absorbed into the bag constant[43]. The masses of the quarks are taken to be: $m_u = 5$ MeV, $m_d = 10$ MeV and $m_s = 200$ MeV.

To know whether there is a phase transition from the nuclear matter to quark matter, we compare the total energies per baryon obtained in the two phases. The EOS corresponding to NM is given by Eqs.(2-3) and that of QM is given by Eq.(29). In Figs.8 and 9 we show the energy per baryon as a function of the baryon density for two values of bag constant, $B^{1/4} = 155$ MeV and $B^{1/4} = 170$ MeV respectively. These are compared with the curves obtained using $\alpha_n = 0.0$ and $\alpha_n = 0.5$ in the same figures. The baryon density ρ_b at which the polarised($\alpha_n = 0.5$) and an unpolarised($\alpha_n = 0.0$) curves intersect is denoted by ρ_{FM}. For the nuclear interaction given in Eq.(1), $\rho_{FM} \simeq 0.405 \, fm^{-3}$. Similarly, the baryon density at which either of the NM curves intersect with the QM one is denoted by ρ_{HQ}.

It can be seen in Fig.8 that for $B^{1/4} = 155$ MeV, the unpolarised NM is energetically favoured upto a density $\rho_b \simeq 0.255 \, fm^{-3}$. As ρ_b is increased further, the quark matter is found to be the lowest energy state. Therefore, there is no region of spin polarised NM in this particular case. On the other hand, for $B^{1/4} = 170$ MeV, it can be seen that for densities within the values $\rho_{FM} \simeq 0.405 \, fm^{-3}$ and $\rho_{HQ} \simeq 1.05 \, fm^{-3}$, the polarised NM is the state of lowest energy. Thus, in this case(Fig.9), one has a spin polarised region

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sandwitched between a quark matter core($\rho_b \geq \rho_{HQ}$) and a unpolarised nuclear matter envelope($\rho_b \leq \rho_{FM}$). Further, we also studied the dependence of the width of the spin polarised region, roughly given by ($\rho_{HQ} - \rho_{FM}$), on the strange quark mass. In Fig.10, we have plotted ($\rho_{HQ} - \rho_{FM})/\rho_{HQ}$ as a function of the bag paramter for three values of m_s. Negative values of ($\rho_{HQ} - \rho_{FM})/\rho_{HQ}$ indicate that there is no region of spin polarised NM inside a hybrid star. In otherwords, to have a spin polarised region sandwitched between an unpolarised NM envelope and a QM core, ρ_{HQ} must be greater than ρ_{FM}. It can be seen that for allowed values of $B^{1/4}$ and m_s, the presence and non-presence of a polarised region are equally probable. Thus, it is clear from the above discussions that the allowed range of B and m_s values is not able to decide upon the presence/non-presence of a spin polarised region in the hybrid star. It would be interesting to know whether the observational limits on the mass and the size of pulsars would help in finding an answer.

In view of this, we explore the structural properties of the hybrid stars choosing three particular values of the bag parameter B while keeping the strange quark mass m_s fixed at 200 MeV. The three different configurations corresponding to the three sets of MIT bag model parameters are shown schematically in Fig.11. It is clear from the figure that for a fixed value of m_s, one can appropriately choose the value of B so that the calculated surface magnetic field is of the order of the observed value($\sim 10^{12}$ G). The mass, size, central density and surface redshift were then calculated for the three values of B. The results obtained pertaining to the maximum mass configuration are given in Table 2. It can be seen that $M_{max}/M_\odot$ and R decreases as the value of B is increased. For $B^{1/4} = 170$ MeV, $M_{max}/M_\odot$ is found to be less than the observed value. Therefore, one may conclude that large regions of spin polarised matter in a hybrid star is ruled out by the observational limit on the mass of pulsars. But, our present study cannot decide upon either of the two configurations obtained with $B^{1/4} = 155$ MeV and $B^{1/4} = 164$ MeV, since the mass of the respective stars and their radii are well within the acceptable limits. We would however like to mention that

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using the proposed equation of state and the MIT bag model with acceptable parameters($B^{1/4} \sim 164$ MeV, m_s=200 MeV), one can in principle describe both the structural properties as well as the surface magnetic field satisfactorily.

6. Summary

To summarise, we have constructed a nuclear equation of state from a finite range momentum and density dependent interaction and then applied it to investigate some properties of spin polarised nuclear matter with particular reference to the neutron star matter. The parameters of the interaction have a firm basis in the well known properties of nuclear matter and of finite nuclei. Extrapolating this interaction to neutron matter and to higher densities, it is found that the present equation of state agrees well with those obtained from more sophisticated calculations.

Introducing spin degrees of freedom, it is seen that at density $\rho \sim 2.5\rho_o$, the neutron star matter undergoes a ferromagnetic phase transition. This aspect is demonstrated by studying the density behaviour of the total energy(Fig.1) and the magnetic susceptibility(Fig.3). To the best of our knowledge, this is the first non-relativistic calculation that gives a ferromagnetic transition at a density well realisable in the neutron star core.

The proposed equation of state is then applied to the investigation of the structure of neutron stars. With increasing polarisation, the maximum mass and the corresponding radius decreases whereas the value of central density increases. The maximum mass of stars and their radii obtained from calculations with $\alpha_n \leq 0.5$ are found to be well in agreement with the observations.

There is a good possibility that one finds quark matter rather than spin polarised nuclear matter at the core of stars. We therefore investigated this plausibility using the MIT bag model. We found that using the proposed EOS and the MIT bag model, one can in principle obtain the maximum mass, size and the surface magnetic field of hybrid stars well within the

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acceptable limits.

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FIGURE CAPTIONS

Fig.1. The energy per particle for pure neutron matter as a function of density is plotted for $\alpha_n = 0$ and $\alpha_n = 0.5$. The calculated results of Friedman-Pandharipande[12] and with Bethe-Johnson potential[33] are also shown.

Fig.2. Velocity of sound v_s obtained in units of c for pure neutron matter taking $\alpha_n = 0.0$, 0.3 and 0.5 is plotted as a function of the density ratio ρ/ρ_o, where $\rho_o = 0.1533$ fm^{-3} is the symmetric nuclear matter saturation density.

Fig.3. Magnetic susceptibility χ of pure neutron matter calculated for two values of nuclear matter incompressibility K is shown as a function of the density ρ, where χ_{free} is the magnetic susceptibility of non-interacting neutron gas.

Fig.4. Beta-equilibrium proton fraction x as a function of density of neutron star matter. In the upper panel, results are shown for $\alpha_n = 0$ with e^- and $e^- + \mu^-$ considered for $\beta-$ equilibrium. In the lower panel, the results for $\alpha_n = 0.3$ and $\alpha_n = 0.5$ are shown with $e^- + \mu^-$.

Fig.5. The neutron star mass is plotted as a function of central density.

Fig.6. The integrated mass upto radius r is plotted as a function of the density at radius r for different spin polarisation α_n.

Fig.7 Moment of inertia obtained using α_n=0.0, 0.3 and 0.5 is shown as a function of density ρ, where $\rho_o = 0.1533$ fm^{-3} is the symmetric nuclear matter saturation density.

Fig.8 The total energy per baryon of quark matter(QM) calculated using the bag model picture is compared with the energies per baryon of

unpolarised($\alpha_n = 0.0$) and polarised($\alpha_n = 0.5$) nuclear matter(NM).

Fig.9 Same as Fig.8, but for $B^{1/4}$=170 MeV.

Fig.10 The width of the spin polarised region, roughly given by $(\rho_{HQ} - \rho_{FM})/\rho_{HQ}$, is plotted as a function of the bag parameter B for three values of strange quark mass, m_s=100, 200 and 300 MeV. Negative values of $(\rho_{HQ} - \rho_{FM})/\rho_{HQ}$ imply that there is no spin polarised region inside a star.

Fig.11 Schematic representation of the three configurations considered in our study of hybrid stars.

TABLE CAPTIONS

Table 1 Values of the mass M_{max}, size R, central density ρ_c, surface redshift z_s and moment of inertia I obtained in the case of neutron stars using three values of α_n are shown. The tabulated values correspond to the maximum mass configuration.

Table 2 Values of the mass M_{max}, size R, central density ρ_c, surface redshift z_s and moment of inertia I obtained in the case of hybrid stars using three values of bag parameter B are shown. The strange quark mass m_s=200 MeV. The tabulated values correspond to the maximum mass configuration.

Table 1

α_n	$M_{max}/M_\odot$	R (km)	ρ_c/ρ_o	z_s
0.0	2.03	10.3	7.6	0.55
0.3	1.89	9.9	8.4	0.52
0.5	1.53	8.7	11.7	0.44

Table 2

$B^{1/4}$ (MeV)	$M_{max}/M_\odot$	R (km)	ρ_c/ρ_o	z_s
155	1.62	10.1	8.8	0.38
164	1.46	9.3	10.3	0.37
170	1.37	8.8	11.4	0.36

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